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Technical Notes

Heat Transfer Behind a Step in High-Enthalpy Laminar Hypersonic Flow

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I. Introduction

THIS Note describes features associated with heat transfer in a separated flow in a high-Mach-number hypersonic flow in low-to-moderate enthalpy range. A simple flow geometry of a rearward facing step (Fig. 1) is considered for analysis. It is assumed that the step height is small and the separating boundary layer is comparable to step height. Under these circumstances, the hypersonic small disturbance theory is assumed applicable for the flow at the step.

Herein, we propose parameters to describe heat transfer behind a step in high-enthalpy laminar hypersonic flow.

The enthalpy range considered was $1.5\ \mathrm{MJ/kg} \leq h_o \leq 26$ and the Mach number range 6 $6 \leq M_\infty \leq 10$. The Reynolds numbers were also moderate, with unit Reynolds numbers being less than 10^6 per meter. It is shown that under these conditions, the flow at the step is dominated mainly by viscous effects and that real gas effects are minimal.

II. Analytical Considerations

The nondimensional wall heat transfer coefficient such as the Stanton Number will depend on a group of dimensionless parameters characteristic of the flow. Expressing therefore, the heat transfer in terms of Stanton number, we may write

$$St(x^*) = f\left[M_\infty, \alpha_\infty, Pr, \gamma, \frac{T_w}{T_o}, Re_L, \frac{\delta}{h}, \frac{\delta}{r_d}\right] \tag{1}$$

where M_∞ and α_∞ are the freestream Mach number and dissociation fraction describing the freestream; Pr and γ are the Prandtl number and the ratio of specific heats describing the fluid medium; T_w/T_o is the wall to stagnation temperature ratio; Re_L is the Reynolds number based on the upstream fetch L; δ/h is the ratio of the boundary-layer thickness at separation to the step height h. The ratio δ/r_d, where r_d is the reaction length scale, can be considered a Damkohler number Ω, equivalent to the rate of diffusion time τ_d in the boundary layer divided by the time of recombination of species in the boundary layer τ_r if the surface is assumed noncatalytic. There is sufficient evidence that in the range of enthalpies considered, the surface catalytic effects are minimal (see for example Hayne [1] and Gai et al. [2]). x^* is the distance downstream of the step normalized by the step height h.

For a given freestream and given fluid medium, one may assume α_∞, Pr, and γ to be constants and in typical experimental conditions pertaining to high-enthalpy hot flow facilities, generally $T_w/T_o \ll 1$. Equation (1) then becomes

$$St(x^*) = f\left[M_\infty, Re_L, \frac{\delta}{h}, \Omega\right] \tag{2}$$

For a laminar boundary layer,

$$\frac{\delta}{h} \sim \frac{L}{h\sqrt{Re_L}}$$

Then,

$$St(x*) = f\left[M_\infty, Re_L, \frac{L}{h\sqrt{Re_L}}, \Omega\right] \tag{3}$$

For complete similarity, the Mach number, the Reynolds number at the step, the parameter $L/h\sqrt{Re_L}$, and the Damkohler number Ω should all be matched. Wallberg [3] suggests that at high-Mach numbers and low-to-moderate Reynolds numbers, when viscous effects dominate, the relevant parameter is the viscous interaction parameter $\bar{V}_\infty$, where

$$\bar{V}_\infty = M_\infty\sqrt{\frac{C_\infty}{Re_L}}$$

where C_∞ is the Chapman–Rubesin constant defined as $\frac{\mu}{\mu_\infty} = C_\infty \frac{T}{T_\infty}$ (Hayes and Probstein [4]).

Assuming that such a similarity holds for the high-enthalpy hypersonic flow over a step and that the step height is small compared with the upstream fetch L over which the boundary layer grows, one can combine $\bar{V}_\infty$ and the well-known hypersonic small disturbance parameter $M_\infty\tau$, where $\tau = h/L$, to take into account the geometric effect of a small step. This assumption will be justified aposteriori by experimental evidence in Sec. III.

The above Stanton number relation can then be written as

$$St(x^*) = f\left[\frac{\bar{V}_\infty}{M_\infty\tau}, \Omega\right] \tag{4}$$

Thus, the heat transfer behind a small step is dependent on three parameters, namely, the viscous interaction parameter $\bar{V}_\infty$, the hypersonic small disturbance parameter $M_\infty\tau$, and any real gas effects through the gas phase Damkohler number Ω. Now, for a frozen flow, $r_d \to \infty$, so that $\Omega \to 0$ and for equilibrium flow $r_d \to 0$ and $\Omega \to \infty$. Then, depending on whether the flow is in complete chemical equilibrium or chemically frozen, the heat transfer will be influenced mainly by viscous effects. Only when $\Omega = \mathcal{O}(1)$, will the nonequilibrium effects become important. In low-to-moderate enthalpy flows (see for example East et al. [5], Gai et al. [6], Mallinson et al. [7], and Hayne et al. [8]), the gas phase Damkohler number Ω is usually of the order of 10^{-2} or less. Even when Ω is as high as 10^{-1}, nonequilibrium effects seem to be minimal as will be shown in discussion of experimental data in Sec. III. We can, therefore, assume, to first order, that the heat transfer behind a step is influenced mainly by viscous effects, so that

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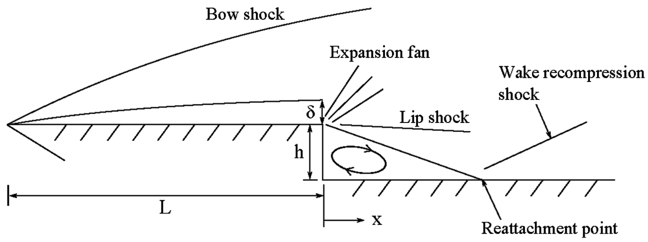

Fig. 1 Schematic of flow behind a rearward facing step.

$$St(x^*) = f\left[\frac{\bar{V}_\infty}{M_\infty \tau}\right] \tag{5}$$

In hypersonic continuum flows over slender bodies, typically, $\bar{V}_\infty \leq 0.1$ (see Potter [9]) and $M_\infty \tau \sim 1$, so that $\frac{\bar{V}_\infty}{M_\infty \tau} \ll 1$. Under typical shock tunnel and expansion tube flow conditions (Gai et al. [10] and Hayne [1]), $0.01 \leq \frac{\bar{V}_\infty}{M_\infty \tau} \leq 0.1$. Note that $\frac{\bar{V}_\infty}{M_\infty \tau}$ becomes identical to the well-known Chapman parameter $(\frac{L}{h\sqrt{Re_L}})$ at moderate-to-high-supersonic Mach numbers and Reynolds numbers and when the hypersonic small disturbance assumption is no longer valid.

III. Comparison with Experiment

The veracity of the relation in Eq. (5) can be examined from the available experimental data, for example Gai et al. [6,10], Hayne [1], and Wada and Inoue [11]. All these experiments dealt with hypersonic laminar flow over a backward facing step. The flow enthalpies (h_o) ranged from 1.5 MJ/kg to 26 MJ/kg, thus covering both undissociated and mildly to moderately dissociated freestreams. The range of the parameter $\frac{\bar{V}_\infty}{M_\infty \tau}$ was $0.0185 \leq \frac{\bar{V}_\infty}{M_\infty \tau} \leq 0.048$. The gas phase Damkohler number Ω ranged from 0 (undissociated flow) to 0.133. Flow conditions and other details are given in Table 1. The results are presented in Figs. 2a and 2b. The data are expressed in terms of the ratio (St/St_{fp}) where the Stanton number behind the step is normalized by the corresponding attached flat plate boundary-layer Stanton number that would exist at the same location in the absence of the step.

For a boundary-layer flow in a perfect gas, the reference enthalpy method (Eckert [12]) in combination with the Reynolds analogy enables the heat transfer rate q_w to be expressed in terms of the Stanton number (see White [13]) as

$$St = 0.332(Pr^*)^{-2/3}(C^*)^{1/2}Re_x^{-1/2} \tag{6}$$

where

$$St = \frac{q_w}{\rho_e u_e (h_r - h_w)}$$

where C^* and Pr^* are evaluated at reference temperature conditions. The recovery enthalpy h_r is defined as

$$h_r = h_o + \frac{u_e^2}{2}(\sqrt{Pr^*} - 1)$$

The subscripts 'w' and 'e' denote wall and edge conditions of the boundary layer and the subscript 'o' denotes stagnation conditions.

In shock tunnel and expansion tube flows, the dissociated freestream may be frozen as a result of nozzle expansion. In these circumstances, part of the freestream total enthalpy may be stored as chemical potential enthalpy. This would then result in reduced heat transfer. In the absence of recombination of the atomic species, either in the boundary layer or at the surface, it has been shown (East et al. [5], Mallinson et al. [7], and Hayne et al. [8]) that the heat transfer may be evaluated using the Eckert [12] reference enthalpy method modified to take into account the effects of freestream dissociation. The expression for the heat transfer in this frozen limit is then

$$St_f = 0.332(C^*)^{1/2}(Pr^*)^{-2/3}Re_x^{-1/2}\left(1 - \frac{h_{\text{chem}}}{h_r - h_w}\right) \tag{7}$$

where h_{chem} is the chemical potential enthalpy due to dissociated species. In the limit of equilibrium boundary-layer chemistry, wherein complete recombination of atomic species takes place within the boundary layer and at the surface, $h_{\text{chem}} = 0$ and the perfect gas relation for heat transfer given by Eq. (6) is restored. The

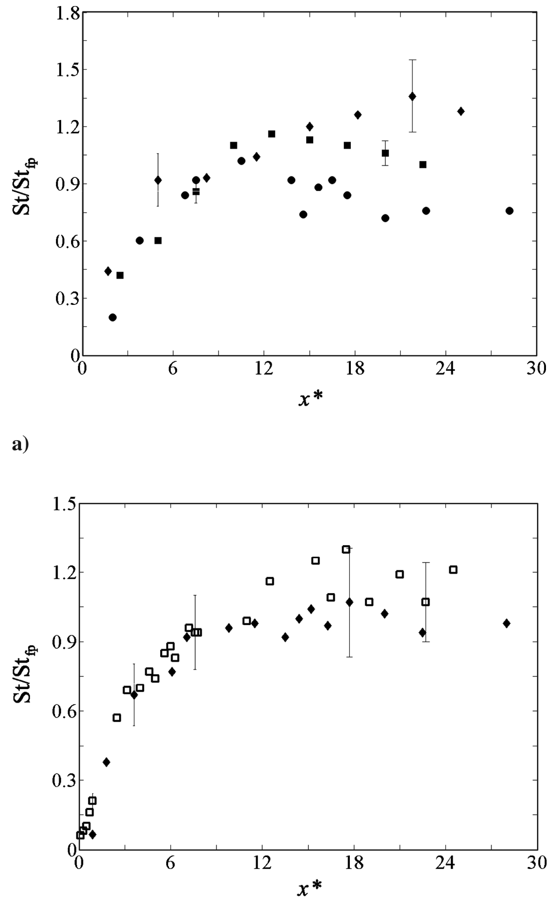

Fig. 2 Stanton number distribution downstream of step: a) ◆, Gai et al. [6], $\frac{\bar{V}_\infty}{M_\infty \tau} = 0.0207$, shock tunnel T3; ■, Wada and Inoue [11], $\frac{\bar{V}_\infty}{M_\infty \tau} = 0.0185$, gun tunnel; and ●, Hayne [1], $\frac{\bar{V}_\infty}{M_\infty \tau} = 0.027$, shock tunnel T4 and b) □, Hayne [1], $\frac{\bar{V}_\infty}{M_\infty \tau} = 0.0456$, expansion tube X2 and ◆, Hayne [1], $\frac{\bar{V}_\infty}{M_\infty \tau} = 0.0478$, shock tunnel T4.

Table 1 Flow and geometric parameters at the step

Reference	h_o, MJ/kg	Re/m	γ	M_∞	$\bar{V}_\infty$	τ	$M_\infty \tau$	$\frac{\bar{V}_\infty}{M_\infty \tau}$	Ω
Gai et al. [6]	2.6	46.6×10^5	1.4	10.1	0.019	0.0909	0.918	0.0207	0.0083
Wada and Inoue [11]	1.5	36×10^5–50×10^5	1.4	10.4	0.016	0.083	0.867	0.0185	0
Hayne [1]	26	11.2×10^5	1.35	7.0	0.029	0.0909	0.636	0.0456	0.133
Hayne [1]	5.4	16.9×10^5	1.38	6.2	0.027	0.0909	0.564	0.0479	0.0569
Hayne [1]	5.6	58.5×10^5	1.37	6.1	0.015	0.0909	0.554	0.027	0.0596

factor $\frac{h_{\text{chem}}}{h_r - h_w}$ thus denotes the fraction by which the heat transfer is reduced from its equilibrium value.

For the experimental data considered herein, the flow is either undissociated or if dissociated, it is chemically frozen. The effects of recombination and dissociation are therefore not significant within the boundary layer. The surface recombination rate is also negligible (Mallinson et al. [7] and Gai et al. [2]) so that we can write

$$St_f = St\left(1 - \frac{h_{\text{chem}}}{h_r - h_w}\right) \tag{8}$$

It is this Stanton number that is used in the experimental results discussed below.

Figure 2a shows the results where the similarity parameter $\frac{\bar{V}_\infty}{M_\infty \tau}$ is nearly the same. The results shown were obtained in three different facilities. Two sets were obtained in two different free-piston driven shock tunnels (T3 and T4) wherein one set of results pertains to an undissociated flow and the other to a mildly dissociated flow. The third set of results was obtained in a hypersonic gun tunnel where the flow was completely undissociated. It is seen that the correlation of heat transfer data is very reasonable considering that the uncertainty in the calculated Stanton number data in high-enthalpy facilities is usually between 15 and 20% (see East et al. [5]). We also note that the agreement is particularly good within about 15 step heights downstream of the step face.

Figure 2b shows the heat transfer data obtained at slightly higher values of the parameter $\frac{\bar{V}_\infty}{M_\infty \tau}$ than in Fig. 2a. These results again were obtained in two different flow facilities, a free-piston driven shock tunnel (T4) and a free-piston driven expansion tube (X2), where the flow enthalpies varied considerably. In the shock tunnel flow ($h_o = 5.4$ MJ/kg), a small fraction of oxygen was dissociated but there was no nitrogen dissociation. In the expansion tube flow ($h_o = 26.0$ MJ/kg), most of the oxygen was dissociated along with a small amount of nitrogen. We again note that agreement is excellent especially in the separation and reattachment regions within about 20 step heights downstream.

The results from Figs. 2a and 2b thus clearly show that the heat transfer behind a step in laminar hypersonic flow is influenced mainly by viscous effects and that $\frac{\bar{V}_\infty}{M_\infty \tau}$ is a suitable parameter to describe the heat transfer distribution. The real gas effects are not significant.

IV. Conclusions

It is shown that within the frame work of hypersonic small disturbance theory, it is possible to correlate the heat transfer distribution behind a small step in laminar hypersonic flow. A new parameter is proposed for such a correlation which shows that the heat transfer is dominated mainly by viscous effects when the gas phase Damkohler number is small and the surface is noncatalytic. Agreement with experimental data in both dissociated and undissociated flows, obtained in different types of flow facilities, is shown to be quite reasonable.

Acknowledgments

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